STUDY OF THE MOTION OF A LIQUID BETWEEN TWO ROTATING SPHERICAL SURFACES

M. P. Sorokin, G. N. Khlebutin, and G. F. Shaidurov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 7, No. 6, pp. 103-104, 1966

In 1961 Bratukhin [1] found that the motion of a liquid between concentric spheres, which arises when the inner sphere rotates and the outer sphere is fixed, becomes unstable when the ratio of the radii $r_2/r_1 = 2$ and the Reynolds number is 100 ($R = r_1^2\omega/\nu$, where ω is the angular velocity of the sphere and ν is the kinematic viscosity of the liquid). The solution was sought in the form of powers of the Reynolds number. Since the validity of this method for $R \sim 100$ is not at all obvious, we have carried out an experiment to verify the theory.

1. The apparatus is shown in Fig. 1. The liquid fills space 1 between a steel nickel-coated sphere of radius $r_1 = 0.945$ cm, which is rotated by a reversing synchronous motor, and a fixed spherical cavity of radius $r_2 = 1.604$ cm cut in a demountable plexiglas cube 2. The cube is freely suspended from a steel wire 3 which is 0.02 cm in diameter and 2.11 m long. The upper end of the wire can be rotated about the vertical axis by means of micrometer screws, and can also be displaced by a known amount both in the vertical and horizontal directions. The spheres can be kept concentric to within 10^{-3} cm, and this is controlled by inspection through transparent windows in the cube using a cathetometer. The length of the working part of the suspension wire could be varied without affecting the concentricity of the spheres by means of grip 4, kept in a paraffin bath. The couple is calculated from the rotation of the wire using a lamp and scale arrangement. Oscillations of the suspension system are damped by an oil damper. The system is placed in water bath 5 whose temperature is kept constant by a thermostat. The upper end is covered by a transparent plexiglas cover 6. Hydrodynamic screening is achieved by means of a fixed thin-walled tube 7. The hot junction of a thermocouple is placed near the tube inside the cavity, and is used to monitor the temperature of the liquid. In addition, hypodermic needles can be used to inject dyes into the liquid for the purposes of visual inspection.

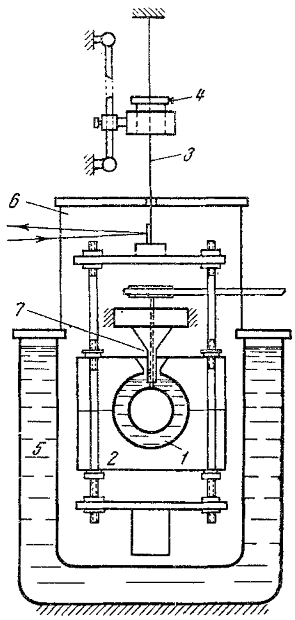

Fig. 1

Experiments were carried out in distilled water and water solutions of technically pure (dynamite) glycerin. The density of the solutions was determined hydrostatically, and the viscosity was found by Stokes' method and with the Ostwald viscometer. The usual corrections were introduced, and the solutions under investigation were carefully thermostated.

2. Measurements showed that the motion of the liquid for all the Reynolds numbers employed did not differ from that described by the second approximation in [1]. The circular horizontal motion had superimposed upon it a less intensive motion in the meridional plane: on the inner sphere the liquid flowed from the poles to the equator, and on the outer sphere it flowed from the equator to the poles.

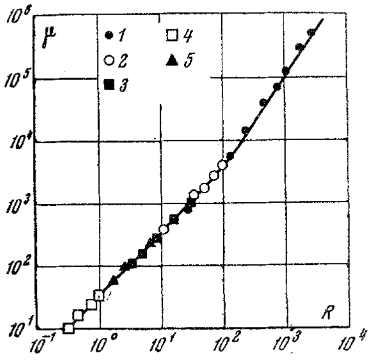

Fig. 2

The velocity of both motions increased with increasing ω. For large ω the colored liquid coated the surfaces of the spheres with a thin layer, and separated from them only on the vertical axis and in the plane of the equator. This suggests the formation of a boundary layer.

Figure 2 shows on a logarithmic scale the results of measurements of the couple M exerted by the liquid on the outer sphere. This figure gives a plot of the Reynolds number in the range 0.7-2800 as a function of the dimensionless couple

$$\mu = \frac{M}{\rho r_1 \nu^2}.$$

Points correspond to the following solutions of glycerin in distilled water: 1) 0%; 2) 45%; 3) 65%; 4) 80%; 5) 85%; the theoretical curve was obtained from Eq. (2.1) for $R < 80$ and from the results reported in [5] for $R > 100$.

The experimental points shown in the figure were selected randomly [2], using tables of random numbers, from a set of 250 measurements. For $R < 80$, the experimental points lie on the theoretical curve

$$\mu = \frac{8\pi r_2^3}{r_2^3 - r_1^3} R, \qquad (2.1)$$

obtained for small Reynolds numbers (see, for example, [3]). For $R > 100$, we have

$$\mu = 3.35\, R^{3/2}. \qquad (2.2)$$

Expressions such as that given by the last equation are obtained in the boundary-layer approximation for a retarding couple on axially symmetric bodies rotating in a liquid [4], and are characteristic for a boundary layer. The numerical coefficient depends on the shape of the body and the volume of the liquid. For a sphere rotating in an infinite medium, the values of the numerical coefficient predicted theoretically in [5-7] are 2.74, 3.04, and 2.56, respectively but, as expected, are somewhat less than our value. No indication that the stability of motion of the liquid was violated was found during our experiments.

In the range of Reynolds numbers where a departure from stability was expected [1] there was continuous crisis-free transition from regularities characteristic for slow motions to boundary-layer type regularities.

We note in conclusion that the results obtained here may be of interest in viscometry.

REFERENCES

1. Yu. K. Bratukhin, "Estimate of the critical Reynolds number for the flow of a liquid between two rotating spherical surfaces," PMM, vol. 25, no. 5, 1961.
2. V. I. Romanovskii, Applications of Mathematical Statistics in Experimental Procedures [in Russian], Gostekhizdat, 1947.
3. N. E. Kochin, I. A. Kibel, and N. V. Roze, Theoretical Hydromechanics, Part 2 [in Russian], Fizmatgiz, 1963.
4. L. A. Dorfman, Hydrodynamic Resistance and Heat Transfer for Rotating Bodies [in Russian], Fizmatgiz, 1960.
5. G. Z. Gershuni and E. M. Zhukhovitskii, "Rotation of a sphere in a viscous conducting fluid in a magnetic field at large Reynolds numbers," Zh. tekhn. fiz., vol. 34, no. 2, 1964.
6. L. Howarth, "Note on the boundary layer on a rotating sphere," Phil. Mag., vol. 42, no. 334, 1954.
7. S. D. Nigam, "Note on the boundary layer on a rotating sphere," Z. angew. Math. und Phys., no. 5, 1954.

6 August 1965 Perm